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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

DMA5101 – MATHEMATICAL TECHNIQUES 1
(All sections / Groups)

12 JULY 2023
9.00 a.m - 11.00 p.m
(2 Hours)

INSTRUCTIONS TO STUDENTS.

1. This question paper consists of 4 pages with **4 Questions** only.
2. Answer **ALL** questions.
3. Please write all your answers in the Answer Booklet provided.
4. All necessary working steps must be shown.

QUESTION 1 (10 marks)

- (a) Simplify the following expression and write your final expression in terms of positive exponents.

$$\frac{\sqrt[3]{x^2} \left(x^{\frac{1}{3}} y^{-2} \right)}{\left(2xy^{-\frac{1}{2}} \right)^2}$$

[1.5 marks]

- (b) Factor the following expression completely. Simplify your answer.

$$\frac{6x - 2}{6x^3 - 2x^2 + 24x - 8}$$

[2 marks]

- (c) Solve the equation $\sqrt{13 - 4x} = 2x + 1$. Verify the correctness of your answer.

[4.5 marks]

- (d) Evaluate the following. Write the final expression in the form of complex number $a + bi$, where $a, b \in \mathbb{R}$.

$$\frac{\sqrt{-16} (1 + \sqrt{-4})}{\sqrt{-1} \sqrt{-12}}$$

[2 marks]**Continued...**

QUESTION 2 (10 marks)

(a) Let functions $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x - 1$.

i. Find $(f \circ g)(2)$.

ii. Find $(g \circ f)(2)$.

[2 marks]

(b) Given $f(x) = \sqrt{x - 6}$.

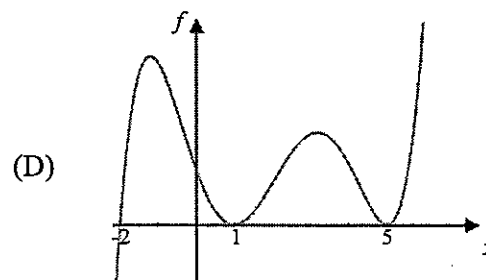
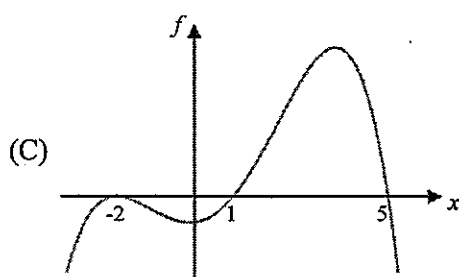
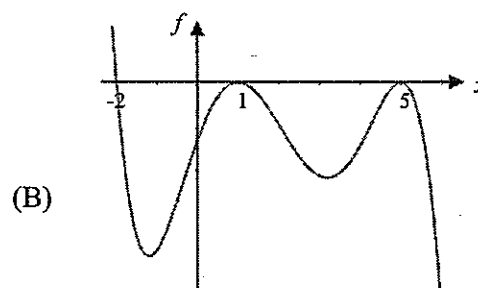
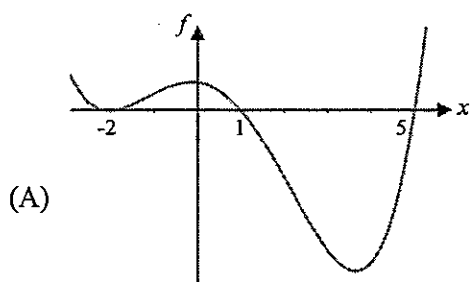
i. State the domain and range of f .

ii. Find $f^{-1}(x)$.

iii. Sketch the graphs of f and f^{-1} on a same xy -coordinate plane. [3.5 marks]

(c) Given a polynomial function $f(x) = -(x + 2)^2(x - 1)(x - 5)$.

- Determine the **zeros** of f and their **multiplicities**. Then, determine whether the graph of f crosses or touches the x -axis at each zero.
- Find the y -intercept.
- Use the **Leading Coefficient Test** to determine the end behavior of f .
- Which of the following is the graph of f ?

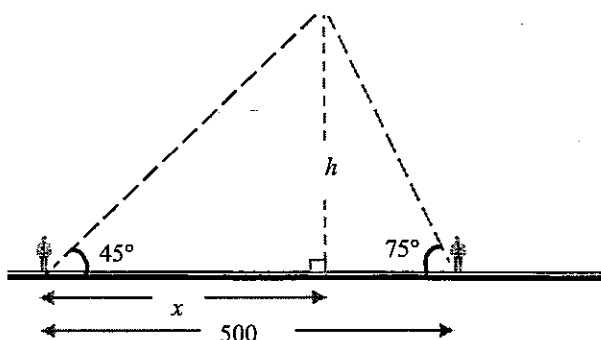


[4.5 marks]

Continued...

QUESTION 3 (10 marks)

- (a) Let $P(-\sqrt{13}, \sqrt{3})$ be a point on the terminal side of angle θ .
- Name the quadrant in which θ lies.
 - Draw point P and angle θ on the xy -coordinate plane.
 - Find $\sin \theta$, $\cos \theta$ and $\tan \theta$. Leave your answers in fraction form. **[4 marks]**
- (b) Two weather observers are assigned to measure the height of the cloud cover at a paragliding base. The first observer shoots a laser beam up into the sky at an angle of 75° from the horizontal ground surface. The second observer 500 meters away from the first observer measures the angle of elevation to the spot of the beam to be 45° . Suppose x be the distance between the second observer and cloud position on the ground. The illustration is given in the figure below.



Find the height h of the cloud cover. Provide your answer correct to 4 decimal places. **[3.5 marks]**

- (c) Let function $y = 3 \sin(\pi x)$.
- Find the amplitude, phase shift, vertical shift and period of y .
 - Then, graph at least one period of y . Label the x -intercept(s) in the graph. **[2.5 marks]**

Continued...

QUESTION 4 (10 marks)

- (a) Use Cramer's rule to solve the linear system.

$$x + y = 3$$

$$2y + z = 0$$

$$x - z = 6$$

[5.5 marks]

- (b) Find the standard form of an equation of a plane that passes through the point $(5, 2, 3)$ and with normal vector $2i - j + 3k$. [2 marks]

- (c) Given two vectors $\mathbf{a} = 2i - j + k$ and $\mathbf{b} = 3i + 2j - k$.

- i. Find dot product $\mathbf{a} \cdot \mathbf{b}$.
- ii. Find $|\mathbf{a}|$.
- iii. Find $|\mathbf{b}|$.
- iv. Find the angle between $\mathbf{a}$ and $\mathbf{b}$. Provide your answer correct to 4 decimal places. [2.5 marks]

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